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Abstract

A survey is presented of several extrema principles of energy dissipation as applied to problems in fluid mechanics. An exact equation is derived for the dissipation function of a homogeneous, isotropic, Newtonian fluid, with terms associated with irreversible compression or expansion, wave radiation, and the square of the vorticity. By using entropy extrema principles, simple flows such as the incompressible channel flow and the cylindrical vortex are identified as minimal dissipative distributions. The principal notions of stability of parallel shear flows appear to be associated with a maximum dissipation condition. These different conditions are consistent with Prigogine's classification of thermodynamic states into categories of equilibrium, linear non-equilibrium, and non-linear non-equilibrium thermodynamics; vortices and acoustic waves appear as examples of dissipative structures. The measurements of a typical periodic shear flow, the rectangular wall jet, show that direct measurements of the dissipative terms are possible.

Nomenclature

a_0	speed of sound in undisturbed fluid
A, B, C	integration constants
δ_{ik}	deformation rate tensor
h	parallel channel height
I_1, I_2, I_3	dissipation integrals (also $I_{1'}$, $I_{2'}$)
k	coefficient of heat conduction
K	vortex strength
L	length of vortex
M	Mach number in the j direction $= u_j/a_0$
p	pressure
r	radial distance
r_c	vortex core radius
t	time
u	streamwise velocity component
$\vec{u}$, u_i	velocity vectors
u_θ	tangential velocity component
U_0	maximum nozzle exit velocity
W	parallel channel width
x	streamwise direction
$\vec{x}$, x_i	position vector
y	direction normal to parallel channel wall
δ_{ik}	Kroneker delta
Φ	viscous dissipation function
Φ_I	incompressible dissipation function
Φ_R	radiative component of dissipation function
λ	second coefficient of viscosity
μ	viscosity
ν	kinematic viscosity
ρ	density
ω, ω_k	vorticity
$\overline{(\ )}$	time average of quantity in parentheses
$(\ )'$	time dependent fluctuation of quantity in parentheses

I. Introduction

The successful application of the "principle of least action" to problems in the fields of particle dynamics and quantum mechanics has inspired several efforts to extend the method to other areas, including thermodynamics and hydrodynamics. The relevance of variational principles to these areas has been studied relatively recently, and hence there is significantly less acceptance of any general framework of extrema principles to them. Several writers [1,2] conclude that variational principles are not applicable to the field of thermodynamics. Onsager examined coupled irreversible processes near equilibrium and derived a statement of minimum entropy production for these conditions [3]. Ziegler [4] proposes that rates of entropy production and energy dissipation are maximized for processes far from equilibrium, while Biot [5] supports Onsager's conjecture of minimum dissipation rate for near-equilibrium processes. Tadmor [6] has also defined a minimum entropy principle in the context of stable numerical solutions of the gas dynamics equations.

In the field of hydrodynamics, Helmholtz [7] proved that flow with negligible inertia is characterized by a minimum of

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viscous dissipation. Hertvel and Lin [1] have derived irrotational and rotational forms respectively of the conservation equations from a variational principle, as related by Yourgrau and Mandelstam [1]. For thermal convection and turbulent flow in channels, Malkus [8,9] proposed a theory based on arguments that the smallest turbulent scales are marginally stable, and that the mean and turbulent characteristics of the flow are determined by the condition that the energy dissipation is maximized. Objections to Malkus's theory have been raised in several areas, among them that extrema of the dissipation function are not generally compatible with conservation laws, and of vagueness in the application of both extremizing conditions and stability criteria. Reynolds and Tiederman [10] found poor agreement between a eddy viscosity simulation of turbulent channel flow and Malkus's prediction of the association of maximum dissipation rate with neutral stability of the mean flow. Nihoul [11] reexamined Malkus's theory with regard to Liapounov stability and described an associated principle of minimum Reynolds number. The problem of convective heat transfer, also considered by Paltridge [12] and Busse [13], can be characterized by maximum heat transfer in some circumstances.

Prigogine [14] has classified entropy production theorems according to a generalized sequence of stable thermodynamic states. Thermodynamic equilibrium, characterized by the absence of gradients of state or kinematic variables, is in a state of maximum entropy and zero entropy production. Linear nonequilibrium processes, studied by Onsager [3], are associated with minimum entropy production. The linear description applies if the generalized thermodynamic fluxes (such as heat flow or fluid deformation) are linearly related to the generalized thermodynamic forces (such as temperature gradient or viscous stress). Many near-equilibrium processes are accurately characterized as linear. Entropy production is not necessarily minimized for nonequilibrium processes which are nonlinear or far from equilibrium. Nonlinearity is generally associated with the generation of chaotic states from initially determinate conditions, such as the transition of laminar flow to turbulence. Prigogine [15,16] has demonstrated that nonlinearity and processes which are far from equilibrium may give rise to global organization from an initially random field, and associates such processes with the evolution of dissipative structures. Prigogine describes several general characteristics of nonlinear systems, such as limit cycles and period-doubling Feigenbaum sequences, which are familiar aspects of high speed and transitional flows. Lugt [17] has commented on the evolution of discrete vortices in shear flows as an example of these dissipative structures.

II. Dissipation functions for a Newtonian fluid

The present discussion examines the dissipation function for a homogeneous Newtonian fluid, which Ziegler [4] writes as

$$\Phi = \lambda d_{ii} d_{jj} + 2\mu d_{jk} d_{jk} \qquad (1)$$

where

$$d_{jk} = \frac{1}{2}\left(\frac{\partial u_j}{\partial x_k} + \frac{\partial u_k}{\partial x_j} \right) \qquad (2)$$

Jeffreys [18] rewrites (1) as:

$$\Phi = \lambda \frac{\partial u_i}{\partial x_i} \frac{\partial u_k}{\partial x_k} + \mu \left[\frac{1}{2} \omega_{jk} \omega_{jk} + 2 \frac{\partial u_j}{\partial x_k} \frac{\partial u_k}{\partial x_j} \right] \qquad (3)$$

where

$$\omega_{jk} = \left(\frac{\partial u_k}{\partial x_j} - \frac{\partial u_j}{\partial x_k} \right)$$

Hinze [19] and Tennekes and Lumley [20] obtain similar decompositions of the time-averaged dissipation function, which are valid for homogeneous turbulence. The last term in (3) appears in several aeroacoustic wave equations, such as that derived by Phillips [21]. Horne and Karamcheti [22] have derived a different formulation of the dissipation function by combining the terms of the equation

$$\nabla \cdot \vec{F} - \frac{DM}{Dt} + M \nabla \cdot \vec{u} = 0 \qquad (4)$$

where:

$$M = \frac{\partial \rho}{\partial t} + u_i \frac{\partial \rho}{\partial x_i} + \rho \frac{\partial u_i}{\partial x_i} \qquad (5)$$

$$F = \rho \frac{\partial u_k}{\partial t} + \rho u_i \frac{\partial u_k}{\partial x_i} + \frac{\partial p}{\partial x_k} - \frac{\partial \tau_{jk}}{\partial x_j} \qquad (6)$$

$$\tau_{jk} = \mu \left(\frac{\partial u_j}{\partial x_k} + \frac{\partial u_k}{\partial x_j} \right) + \delta_{jk} \lambda \frac{\partial u_i}{\partial x_i}$$

$M = F = 0$ for a fluid free of sources of mass or momentum. By expanding the terms of equation (4), we obtain

$$\nabla \cdot \vec{F} = \boxed{A}\ \frac{\partial \rho}{\partial x_k} \frac{\partial u_k}{\partial t} + \boxed{B}\ \rho \frac{\partial}{\partial t}\left(\frac{\partial u_k}{\partial x_k} \right) + \boxed{C}\ \frac{\partial p}{\partial x_k} u_i \frac{\partial u_k}{\partial x_i}$$

$$+ \rho \frac{\partial u_i}{\partial x_k} \frac{\partial u_k}{\partial x_i} + \boxed{D}\ \rho u_i \frac{\partial}{\partial x_i}\left(\frac{\partial u_k}{\partial x_k} \right) + \frac{\partial^2 p}{\partial x_k \partial x_k} - \frac{\partial^2 \tau_{jk}}{\partial x_j \partial x_k}$$

$$- \frac{\partial M}{\partial t} = - \frac{\partial^2 \rho}{\partial t^2} - \boxed{A}\ \frac{\partial u_i}{\partial t} \frac{\partial \rho}{\partial x_i} - u_i \frac{\partial}{\partial t}\left(\frac{\partial \rho}{\partial x_i} \right) - \boxed{E}\ \frac{\partial \rho}{\partial t} \frac{\partial u_i}{\partial x_i}$$

$$- \boxed{B}\ \rho \frac{\partial}{\partial t}\left(\frac{\partial u_i}{\partial x_i} \right)$$

$$- u_k \frac{\partial M}{\partial x_k} = - u_k \frac{\partial}{\partial x_k}\left(\frac{\partial \rho}{\partial t} \right) - \boxed{C}\ u_k \frac{\partial u_i}{\partial x_k} \frac{\partial \rho}{\partial x_i} - u_i u_k \frac{\partial^2 \rho}{\partial x_i \partial x_k}$$

$$- \boxed{F}\ u_k \frac{\partial \rho}{\partial x_k} \frac{\partial u_i}{\partial x_i} - \boxed{D}\ \rho u_k \frac{\partial}{\partial x_k}\left(\frac{\partial u_i}{\partial x_i} \right)$$

$$M(\nabla \cdot \vec{u}) = \boxed{E}\ \frac{\partial \rho}{\partial t}\left(\frac{\partial u_k}{\partial x_k} \right) + \boxed{F}\ u_i \frac{\partial \rho}{\partial x_i} \frac{\partial u_k}{\partial x_k} + \rho \frac{\partial u_i}{\partial x_i} \frac{\partial u_k}{\partial x_k}$$

By cancelling terms as shown, the remaining terms in equation (4) appear as follows:

$$\frac{\partial u_i}{\partial x_k}\frac{\partial u_k}{\partial x_i} = +\frac{1}{\rho}\frac{\partial^2 \tau_{ik}}{\partial x_i \partial x_k} - \frac{\partial u_i}{\partial x_i}\frac{\partial u_k}{\partial x_k} \qquad (7)$$

$$-\frac{1}{a^2}\left(\frac{\partial^2 p}{\partial x_k \partial x_k} - u_i u_k \frac{\partial^2 \rho}{\partial x_i \partial x_k} - 2 u_k \frac{\partial^2 \rho}{\partial x_k \partial t} - \frac{\partial^2 \rho}{\partial t^2}\right)$$

$$\frac{1}{\rho}\frac{\partial^2 \tau_{ik}}{\partial x_i \partial x_k} = \frac{(\lambda+2\mu)}{\rho}\frac{\partial^2}{\partial x_k \partial x_k}\left(\frac{\partial u_i}{\partial x_i}\right)$$

By substituting equation (7) into equation (3), the desired form is obtained:

$$\Phi = \frac{2\mu}{\rho}(2\mu+\lambda)\frac{\partial^2}{\partial x_k \partial x_k}\left(\frac{\partial u_i}{\partial x_i}\right) + (\lambda - 2\mu)\frac{\partial u_i}{\partial x_i}\frac{\partial u_k}{\partial x_k}$$

$$\frac{2\mu}{\rho}\left(\frac{\partial^2 p}{\partial x_k \partial x_k} - u_i u_k \frac{\partial^2 \rho}{\partial x_i \partial x_k} - 2 u_k \frac{\partial^2 \rho}{\partial x_k \partial t} - \frac{\partial^2 \rho}{\partial t^2}\right)$$

$$-\frac{\mu}{2}\omega_k \omega_k \qquad (8)$$

This exact expression accounts for three important mechanisms of energy dissipation in real flows: (1) irreversible expansion or compression, (2) generation and radiation of sound or shock waves, and (3) generation of vorticity. Since the mass and momentum conservation equations have been used in the derivation of (8), extrema of this function will satisfy those equations. Formally, functions of the conservation equations have been adjoined to the conventional expression for the dissipation function similar to the manner of imposing functional constraints in the solution of problems of geodesics. The radiative terms can be further manipulated to yield a convective wave operator acting on the density field:

$$\Phi_r = \frac{2\mu a_o^2}{\rho}\left\{\frac{\partial^2}{\partial x_k \partial x_k}\left(\rho - \frac{p}{a_o^2}\right)\right.$$

$$\left.-\left(\frac{\partial^2 \rho}{\partial x_k \partial x_k} - M_i M_k \frac{\partial^2 \rho}{\partial x_i \partial x_k} - 2\frac{M_k}{a_o}\frac{\partial^2 \rho}{\partial x_k \partial t} - \frac{1}{a_o^2}\frac{\partial^2 \rho}{\partial t^2}\right)\right\}$$

where M_k is the Mach number in the k direction. The first term occurs in Lighthill's [24] acoustic stress tensor, while the convective wave operator for the density is identical to the convective wave operator for acoustical disturbances in shear flow. The presence of non-dissipative outward radiating acoustic waves from an aerodynamic source region requires dissipation of energy from the source region.

III. Entropy considerations in low-speed parallel and cylindrical shear flows

For incompressible, uniform-density flows, the expression (8) for the dissipation function reduces to:

$$\Phi_i = -\frac{2\mu}{\rho_o}\nabla^2 p + \mu\omega^2 \qquad (9)$$

This expression may be used to examine low-speed flows for velocity distributions which result in extrema of the dissipation rate. For parallel channel flow, the mass and momentum equations reduce to:

$$\frac{\partial u}{\partial x} = 0$$

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + \frac{\mu}{\rho_0}\frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial p}{\partial y} = 0$$

For this case, the term in the dissipation function associated with the Laplacian of the pressure is zero, as is seen by taking the x-derivative of the momentum equation and by using the continuity equation:

$$\frac{\partial}{\partial x}\left(\frac{\partial \rho}{\partial t}\right) = -\frac{1}{\rho_0}\frac{\partial^2 p}{\partial x^2} + \frac{\mu}{\rho_0}\frac{\partial^2}{\partial y^2}\left(\frac{\partial \rho}{\partial x}\right)$$

Therefore,

$$\frac{\partial^2 p}{\partial x^2} = \frac{\partial^2 p}{\partial y^2} = 0$$

The volume integral of the dissipation function is now given by:

$$\int_v \Phi_i \, dv = \mu \int_v \omega^2 \, dv = \mu x w \int_{y_1}^{y_2}\left(\frac{\partial u}{\partial y}\right)^2 dy \qquad (10)$$

where x and w are the length and width, respectively, of the channel. The flow rate through the channel is given by:

$$V_c = \int_{y_1}^{y_2} u \, dy$$

Possible extrema of the integral (10), subject to constant channel flow rate, are found as solutions of the Euler-Lagrange equation:

$$\frac{\partial f}{\partial u} - \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial u'}\right) = 0, \qquad u' = \frac{\partial u}{\partial y}$$

where:

$$f = \left(\frac{\partial u}{\partial y}\right)^2 + \alpha u$$

Here, α is a Lagrange multiplier adjoining the integrand of the flow rate constraint to the extremizing function. For this example, evaluation of the Euler-Lagrange equation yields the expression:

$$\alpha = 2\frac{\partial^2 u}{\partial y^2}$$

Hence, an extremum of the dissipation is found for the linear-parabolic profile:

$$u(y) = Ay^2 + By + C$$

This profile corresponds to a minimum of the dissipation function, according to Legendre's test, since:

$$\frac{\partial^2 f}{\partial u\, \partial u'} = +2$$

In general, a potential analysis method for other configurations would proceed by extremizing the dissipation function subject to satisfaction of a constraint or set of constraints such as an integral of the flow rate, momentum, or energy through a given section.

Another simple example is that of a steady cylindrical rotational flow, in which the tangential velocity and pressure are functions only of the radius from the origin. Hence:

$$\bar{u} = \{u_r, u_\theta, u_z\};\ u_r = u_z = 0$$

The radial pressure gradient is given by the radial momentum equation:

$$\frac{1}{\rho}\frac{\partial p}{\partial r} = \frac{u_\theta^2}{r}$$

The Laplacian of the pressure is written as:

$$\nabla^2 p = \frac{1}{r}\frac{d}{dr}\left(r\frac{dp}{dr}\right) = \frac{1}{r}\frac{d}{dr}\left(\rho u_\theta^2\right) \qquad (11)$$

The vorticity is given by:

$$\omega = \nabla \times \bar{u} = \frac{1}{r}\frac{d}{dr}\left(r u_\theta\right) = \frac{du_\theta}{dr} + \frac{u_\theta}{r} \qquad (12)$$

$$\omega^2 = \left(\frac{du_\theta}{dr}\right)^2 + \frac{2}{r}u_\theta\frac{du_\theta}{dr} + \frac{u_\theta^2}{r^2} \qquad (13)$$

We seek extrema of the integral:

$$\int_v \Phi\, dv = -\frac{2\mu}{\rho} 2\pi L \int_0^\infty \nabla^2(p)\, r\, dr + \mu 2\pi L \int_0^\infty \omega^2 r\, dr$$

where L is the length of the vortex. The first integral on the right side vanishes, since from equation (11):

$$\int_0^\infty \nabla^2(p)\, r\, dr = \int_0^\infty \frac{1}{r}\frac{d}{dr}\left(u_\theta^2\right) r\, dr = u_\theta^2\Big|_0^\infty$$

and

$$u_\theta(0) = u_\theta(\infty) = 0$$

for any realizable flow. Then:

$$\int_v \Phi\, dv = 2\pi L \int_0^\infty \left[\left(\frac{du_\theta}{dr}\right)^2 + \frac{2}{r}u_\theta\frac{du_\theta}{dr} + \frac{u_\theta^2}{r^2}\right] r\, dr \qquad (14)$$

Extrema of this integral are obtained as solutions of the Euler-Lagrange equation:

$$\frac{\partial g}{\partial u_\theta} - \frac{d}{dr}\left(\frac{\partial g}{\partial u'_\theta}\right) = 0; \quad u'_\theta = \frac{du_\theta}{dr} \qquad (15)$$

$$g = r\left(\frac{du_\theta}{dr}\right)^2 + 2u_\theta\frac{du_\theta}{dr} + \frac{u_\theta^2}{r}$$

$$\frac{\partial g}{\partial u_\theta} = 2\frac{du_\theta}{dr} + 2\frac{u_\theta}{r} \qquad (16)$$

Then,

$$\frac{\partial g}{\partial u'_\theta} = 2r\frac{du_\theta}{dr} + 2u_\theta$$

$$\frac{d}{dr}\left(\frac{\partial g}{\partial u'_\theta}\right) = 2\frac{du_\theta}{dr} + 2r\frac{d^2u_\theta}{dr^2} + 2\frac{du_\theta}{dr} \qquad (17)$$

By substituting equations (16) and (17) into (15), we obtain:

$$r^2\frac{d^2u_\theta}{dr^2} + r\frac{du_\theta}{dr} - u_\theta = 0$$

This is recognized as Euler's equation, with solutions:

$$u_\theta = r,\ r^{-1}$$

The same solutions result if the pressure term is retained. These two solutions represent respectively the core and the outer-potential region of a columnar vortex:

$$u_\theta = Kr, \quad 0 < r < r_0$$

$$u_\theta = \frac{Kr_0^2}{r}; \quad r_0 < r < \infty$$

Both solutions represent minimum dissipation conditions, as in the channel flow. Integrated contributions to the overall dissipation rate are given in the following table:

region	$\iint -2\frac{\mu}{\rho}\nabla^2 p\, r\, dr\, d\theta$	$\iint \mu\omega^2 r\, dr\, d\theta$
$0 < r < r_0$	$-4\pi\mu K^2 r_0^2$	$+4\pi\mu K^2 r_0^2$
$r_0 < r < \infty$	$+4\pi\mu K^2 r_0^2$	0

As can be seen, the net dissipation in the rotational core is zero, while the irrotational outer flow has a net positive dissipation due to the Laplacian of the pressure. The tangential acceleration is zero throughout the flow, except at the boundary at r_0. A steady tangential stress is required at this location (as provided by a thin rotating cylinder, for example) to maintain a steady flow. The net dissipation in the outer region is identical to the torque power absorbed by the rotating cylinder, since the shear stress at the cylinder wall is given by:

$$\sigma = \mu\left(\frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right)$$

The two example considered are steady flows, since the Euler-Lagrange equations did not include time dependent variations. The two examples also involve no convective accelerations, and therefore do not demonstrate an extension of the minimum dissipation principle beyond Helmholtz's original statement for flows with negligible inertial forces. For these flows, the minimum dissipation condition appears to be consistent with a linear, near-equilibrium process. It should be possible to extend the method to boundary layers and asymmetric flows where convective accelerations are important.

IV. Dissipation considerations in unsteady, two-dimensional flows

The application of extremum conditions of entropy production or dissipation rate to unsteady flows may be facilitated by considering the mean and fluctuating components of the dissipation function:

$$p = \bar{p} + p' ; \quad \omega = \bar{\omega} + \omega'$$

$$\Phi_1 = -\frac{2\mu}{\rho}\nabla^2(\bar{p} + p') + \mu(\bar{\omega} + \omega')^2$$

$$\bar{\Phi}_1 = -\frac{2\mu}{\rho}\nabla^2\bar{p} + \mu\left[(\bar{\omega})^2 + \overline{(\omega')^2}\right] \qquad (18)$$

We may use this expression to interpret the small and large disturbance motions in two-dimensional flow. A large number of parallel shear flows such as the boundary-layer, the free-shear-layer, and the wall-jet have been successfully analyzed for stability to small disturbances via the Orr-Sommerfield equation. The stability characteristics for these flows are found to be strongly associated with the mean velocity profile of the flow. For velocities which exceed a critical Reynolds number, small disturbances of the frequency corresponding to the maximum amplification rate are predicted to grow exponentially with downstream distance until nonlinear effects limit the growth. Within the small-disturbance region, the mean velocity profile remains unchanged, and the small disturbances take the form of convecting vortical motions. Experimental studies of various flows confirm that the frequencies predicted for maximum-vortical-disturbance growth rate correspond to the observed frequencies of unforced fluctuations. From equation (18), it appears that the maximum vortical-disturbance growth rate condition corresponds to a maximum dissipation condition, if the contribution from the pressure term is negligible and the contribution from the mean vorticity term is independent of disturbance frequency.

Figure 1 illustrates the periodic vortex motion of a two-dimensional wall jet flow. The wall jet develops from an initially parabolic profile rather than a self-similar profile, as is the case in the parallel shear flow. Woolley and Karamcheti [25] have shown that the stability characteristics of nonparallel shear flows are closely related to those of parallel shear flows. The short wall was associated with the generation of tones by the jet, which significantly reduced the small disturbance region by effectively forcing the initial jet region, however the observed tone frequencies were approximately the same as with the long-wall, silent jet. The conditions for this jet are: U_m = 13.85 m/s, L/h = 7.5, where U_m = the maximum velocity of the parabolic exit profile, L = wall length, and h = nozzle width. Figure 1(a) shows a phase-averaged Schlieren visualization record obtained by lightly heating the subsonic nozzle flow. Figure 1(b) shows the phase-averaged velocity field, referenced to the convecting vortices, and Fig. 1(c) depicts the corresponding vorticity field obtained with a central-differenced curl of the velocity field. The vorticity is normalized with respect to the maximum exit velocity and the nozzle width. The measurements were obtained from a single x-wire velocity probe by sampling the probe output at regular phase intervals as determined by a fixed pressure-transducer in the wall [23]. These measurements were further processed to obtain estimates of the mean and fluctuating components of the dissipation field.

Relative contributions to the dissipative structure of the wall jet were estimated by integrating the field variables in the cross-stream direction. We define:

$$I_T\left(\frac{x}{h}\right) = \left(\frac{h}{U_0}\right)^2 \int \frac{\Phi}{\mu}\, d\left(\frac{y}{h}\right) = I_1 + I_2 + I_3$$

$$I_1 = \left(\frac{h}{U_0}\right)^2 \int -\frac{2}{\rho}\nabla^2 p\, d\left(\frac{y}{h}\right)$$

$$I_2 = \left(\frac{h}{U_0}\right)^2 \int \bar{\omega}^2\, d\left(\frac{y}{h}\right)$$

$$I_3 = \left(\frac{h}{U_0}\right)^2 \int (\omega')^2\, d\left(\frac{y}{h}\right)$$

The Laplacian of the pressure was computed from the divergence of the incompressible, viscous, 2-D momentum equation:

$$-\frac{2}{\rho}\nabla^2 p = 2\left(\frac{\partial u}{\partial x}\right)^2 + 4\left(\frac{\partial u}{\partial y}\right)\left(\frac{\partial v}{\partial x}\right) + 2\left(\frac{\partial v}{\partial y}\right)^2$$

In order to estimate the measurement error, the mean square of the divergence of the velocity was computed:

$$I_{div} = \left(\frac{h}{U_0}\right)^2 \int\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)^2 d\left(\frac{y}{h}\right)$$

Figure 2 shows the distributions of I_1, I_2, I_3, I_{div}, and I_T for the jet conditions described previously. The measurement error indicated by I_{div} is negligible only upstream of the wall trailing edge, where the pressure Laplacian term, I_1, is also negligible. The square of the mean vorticity, I_2, is nearly constant in the small disturbance region: $0 < x/h < 2.5$, beyond which it gradually decreases. The mean square of fluctuating vorticity, I_3, is negligible in the initial region, then steadily increases in the large disturbance wall region: $2.5 < x/h < 7.5$. Relative comparisons of the magnitudes of the pressure and vorticity fluctuation terms in the small disturbance region are not possible with these measurements, but could be made from a stability analysis of the flow.

These measurements demonstrate a potential method of directly measuring dissipative terms in an unsteady flow. A direct extension of this method may be applied to experimentally determine a relationship between overall dissipation and variable parameters, such as forcing frequency, and to experimentally search for dissipation extrema.

V. Conclusions

An exact equation has been derived for the dissipation function of a homogeneous, isotropic, Newtonian fluid, with terms associated with irreversible compression or expansion, wave radiation, and the square of the vorticity. Simple flows such as the incompressible channel flow and the cylindrical vortex are identified as minimal dissipative distributions. The principal notions of stability of parallel shear flows appear to be consistent with maximum dissipation conditions on the growth of vortical disturbances. These observations are consistent with Prigogine's [14,15] distinction between stable thermodynamic states or processes which are either linear and near equilibrium, or nonlinear and far from equilibrium. In this context, vortices and acoustic wave sources appear to be examples of dissipative structures. The measurements of a typical periodic shear flow, the rectangular wall jet, show that direct measurements of the dissipative terms are possible.

Further experimental and theoretical analyses are required to determine the global validity of extrema principles with regard to complex flows and to determine the applicability of time-averaged analyses.

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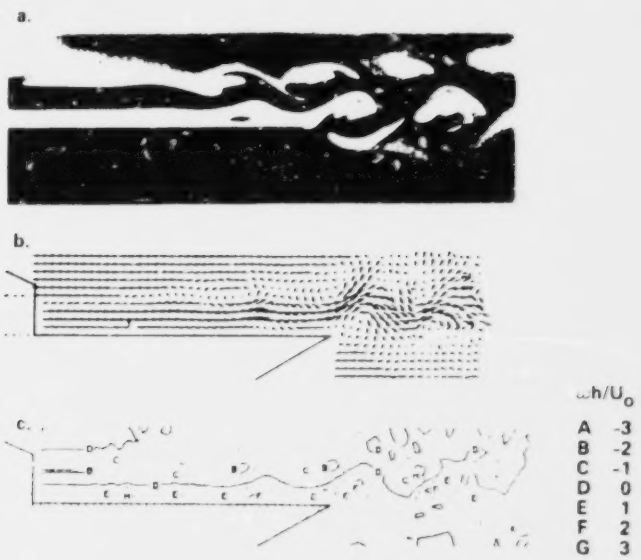

Fig. 1 Phase averaged velocity field and flow visualization. a) Phase-averaged flow visualization; b) velocity vectors (relative to convecting vortices; and c) vorticity contours. Conditions: nozzle width, h = 0.508 cm; nozzle aspect ratio = 20; wall length, L = 3.81 cm; parabolic velocity profile at nozzle exit with maximum exit velocity, U_o = 13.85 m/sec; tone frequency, f = 600 Hz.

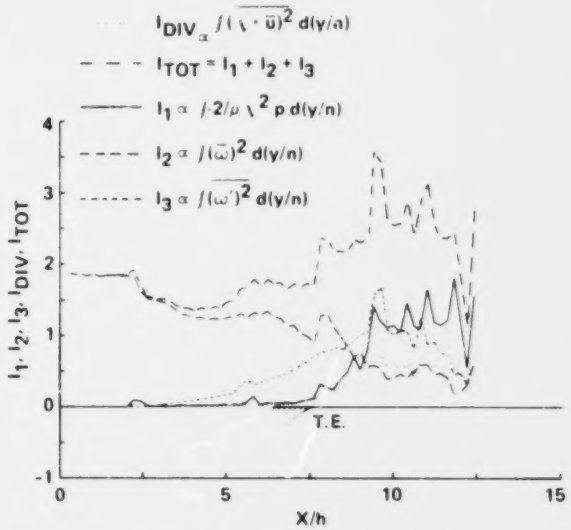

Fig. 2 Distribution of integrated dissipation components.

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16. Abstract

A survey is presented of several extrema principles of energy dissipation as applied to problems in fluid mechanics. An exact equation is derived for the dissipation function of a homogeneous, isotropic, Newtonian fluid, with terms associated with irreversible compression or expansion, wave radiation, and the square of the vorticity. By using entropy extrema principles, simple flows such as the incompressible channel flow and the cylindrical vortex are identified as minimal dissipative distributions. The principal notions of stability of parallel shear flows appears to be associated with a maximum dissipation condition. These different conditions are consistent with Prigogine's classification of thermodynamic states into categories of equilibrium, linear non-equilibrium, and non-linear non-equilibrium thermodynamics; vortices and acoustic waves appear as examples of dissipative structures. The measurements of a typical periodic shear flow, the rectangular wall jet, show that direct measurements of the dissipative terms are possible.

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